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On the Radiation from Microstrip Discontinuities

MOHAMAD D. ABOUZAHRA, STUDENT MEMBER, IEEE

***Abstract*—The method of an earlier paper by Lewin is used to calculate, more accurately, the radiated power from a microstrip termination. The substrate dielectric constant ϵ is used instead of the effective dielectric constant ϵ_e in the polarization term. The open-circuit, short-circuit, and matched coaxial terminations are deduced as particular cases of the general termination. On comparison with Lewin's results, differences of up to 30 percent have been found, but the differences are much smaller for the larger values of the actual relative dielectric constant ϵ. Curves show that the short-circuit termination radiates less than a quarter that of the open circuit, and can be considered as a means of reducing losses in microstrip resonators. The parallel post configuration is also considered.**

I. INTRODUCTION

IN AN earlier paper [1], Lewin used the far-field Poynting vector method to calculate the radiation from microstrip discontinuities. In the course of his calculation, and in order to account for the leakage of the field into the air above the strip, Lewin used the effective relative dielectric constant ϵ_e in both the propagation constant and the polarization part of the calculation. Using a completely different type of analysis, utilizing Fourier transforms and a more involved treatment of the microstrip configuration, Van der Pauw [2] derived a more accurate expression for the open-circuit case. Recently, the calculations of Lewin for the open-circuit and matched termination were repeated [3] with ϵ_e replaced by ϵ in the polarization term. The results for the open-circuit case agree with that of Van der Pauw. From [3] it was discovered that the main difference between the results of [1] and [2] was not from the radically different treatment, but from the use of ϵ_e rather than ϵ in the calculation of the contribution of the dielectric polarization to the radiated fields. In this paper, Lewin's method will be extended to derive a more accurate expression for the general termination case from which results for three particular cases will be deduced. These are the open-circuit, short-circuit, and the matched coaxial termination. The parallel post configuration is also reconsidered and a more accurate expression for the radiated fields and the radiated power are derived.

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The author is with the Electromagnetics Laboratory, Department of Electrical Engineering, University of Colorado, Boulder, CO 80309.

II. ANALYSIS

Fig. 1 shows the microstrip configuration as well as the coordinate system used in this paper. In this analysis, a new scheme of notation, different from that of [3], will be adopted. By replacing ϵ_e (ϵ in [3]), the *effective* dielectric constant, by ϵ (ϵ^* in [3]), the *actual* dielectric constant, in the polarization term and then calculating the far-field Hertzian vector, the far-field expressions for the mis-

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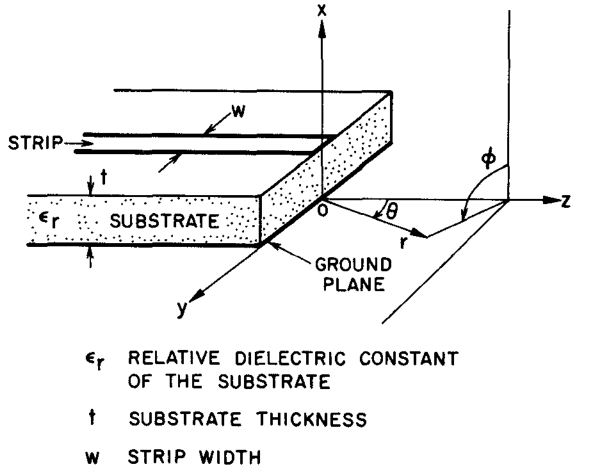

Fig. 1. Microstrip configuration and coordinate system.

matched microstrip termination take the form

$$E_\phi = j\frac{60kt}{\epsilon}\left[\frac{\epsilon\cos\theta-\sqrt{\epsilon_e}}{\sqrt{\epsilon_e}-\cos\theta} - \mathrm{Re}^{j\psi}\frac{\sqrt{\epsilon_e}+\epsilon\cos\theta}{\sqrt{\epsilon_e}+\cos\theta}\right]\sin\phi\frac{e^{-jkr}}{r} \tag{1}$$

$$E_0 = -j\frac{60kt}{\epsilon}\left[\frac{\sqrt{\epsilon_e}(\epsilon-\cos^2\theta)+(\epsilon-\epsilon_e)\cos\theta}{\epsilon_e-\cos^2\theta} + \mathrm{Re}^{j\psi}\frac{(\epsilon-\epsilon_e)\cos\theta-\sqrt{\epsilon_e}(\epsilon-\cos^2\theta)}{\epsilon_e-\cos^2\theta}\right]\cos\phi\frac{e^{-jkr}}{r} \tag{2}$$

where $\mathrm{Re}^{j\psi}$ represents the current reflection coefficient of the termination.

The Poynting vector was calculated for a unit incident current wave, and integrated over an infinite hemisphere to give the radiated power as

$$P = 60(kt)^2 F(\epsilon_e, \epsilon) \tag{3}$$

where $k = 2\pi/\lambda_0$, t is the substrate thickness, and $F(\epsilon_e, \epsilon)$ is the radiation factor. $F(\epsilon_e, \epsilon)$ depends on the discontinuity and is found to be

$$F_0(\epsilon_e,\epsilon) = \frac{\epsilon_e(\epsilon^2+\epsilon_e-2\epsilon)}{\epsilon^2(\epsilon_e-1)}(1+R^2) + R\cos\psi\left(1-\frac{\epsilon_e}{\epsilon^2}\right) - \frac{\sqrt{\epsilon_e}}{2\epsilon^2}\left[(\epsilon^2+\epsilon_e-2\epsilon)(1+R^2) + (\epsilon^2-\epsilon_e)\left(1+\frac{1}{\epsilon_e}\right)R\cos\psi\right]\log\frac{\sqrt{\epsilon_e}+1}{\sqrt{\epsilon_e}-1} \tag{4}$$

for a mismatched terminated microstrip end.

When $R = 1$ and $\psi = \pi$ the arrangement is an open circuit and (1), (2), and (4) reduce to (4), (5), and (6) of [3]. Similarly, when $R = 0$ the mismatched stripline reduces to the matched arrangement, and the expressions reduce to (8), (9), and (10), respectively of [3].

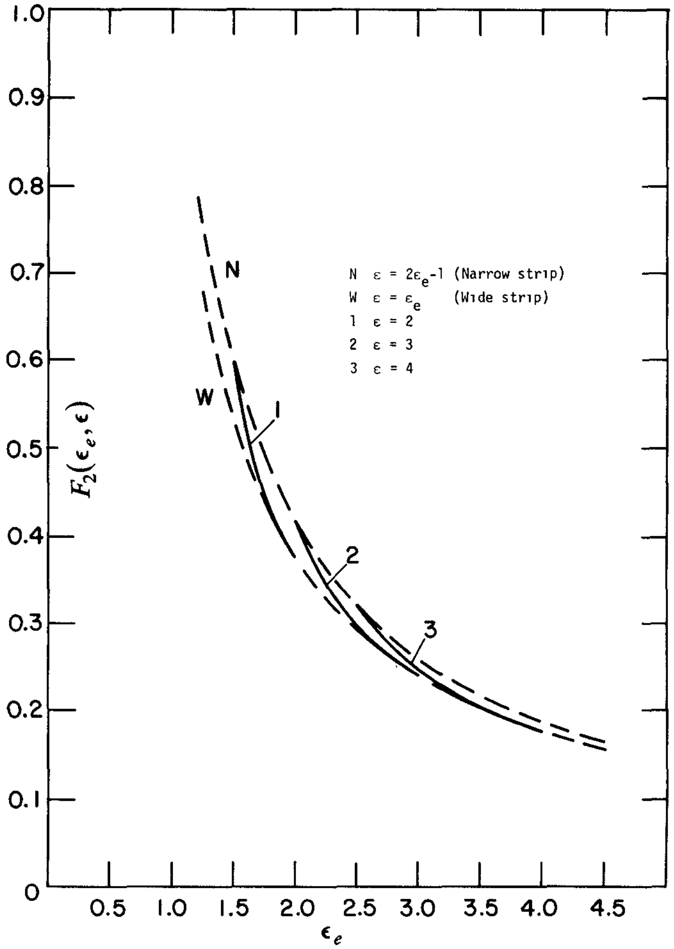

Fig. 2. Radiation factor of a matched termination; $F_2(\epsilon_e, \epsilon)$ versus ϵ_e.

Fig. 2 is a graph for $F_2(\epsilon_e, \epsilon)$ (the radiation factor of the matched termination) versus ϵ_e for a range of ϵ. The values of ϵ_e range from ϵ for very wide strips to $(\epsilon+1)/2$ for very narrow strips [4]. It is clearly seen that the modified formula, which takes the form

$$F_2(\epsilon_e,\epsilon) = \frac{\epsilon_e(\epsilon^2+\epsilon_e-2\epsilon)}{\epsilon^2(\epsilon_e-1)} - \frac{\sqrt{\epsilon_e}}{2\epsilon^2}(\epsilon^2+\epsilon_e-2\epsilon)\log\frac{\sqrt{\epsilon_e}+1}{\sqrt{\epsilon_e}-1} \tag{5}$$

gives a greater value than the original (Lewin's equation (15)) with differences up to 10 percent. However, in agreement with the open-circuit arrangement, the differences decrease greatly for large values of ϵ_e.

Upon substituting $R = 1$ and $\psi = 0$ in (1), (2), and (4), the general termination case reduces to the short-circuit arrangement. The far-zone electric-field components become

$$E_\phi = j\frac{120kt}{\epsilon}\left[\frac{\epsilon\cos^2\theta-\epsilon_e}{\epsilon_e-\cos^2\theta}\right]\sin\phi\frac{e^{-jkr}}{r} \tag{6}$$

$$E_\theta = -j\frac{120kt}{\epsilon}\left[\frac{(\epsilon-\epsilon_e)\cos\theta}{\epsilon_e-\cos^2\theta}\right]\cos\phi\frac{e^{-jkr}}{r} \tag{7}$$

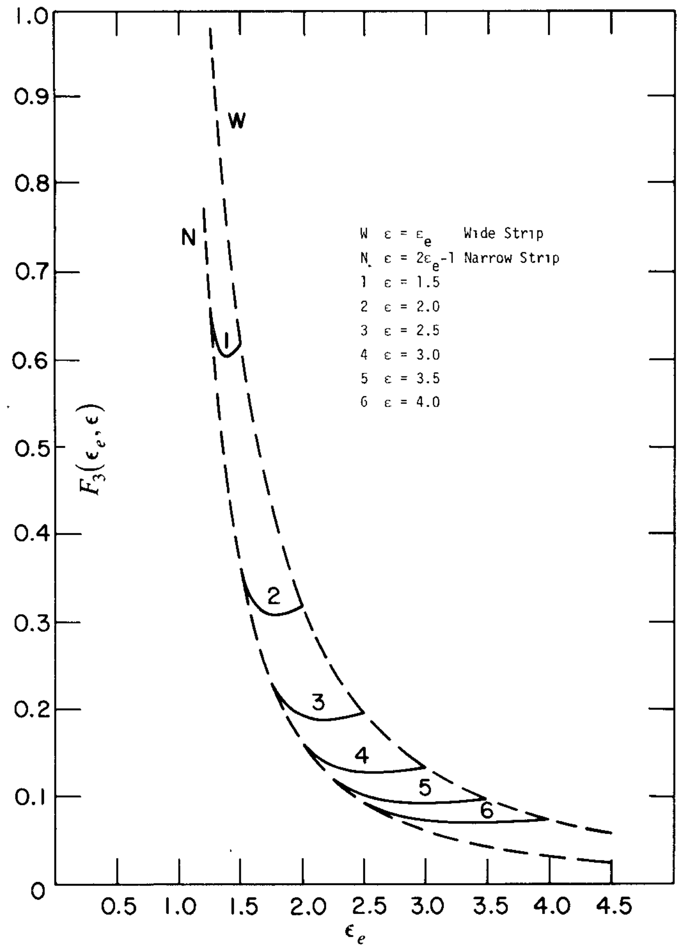

Fig. 3. Radiation factor of a short-circuit termination; $F_3(\epsilon_e, \epsilon)$ versus ϵ_e.

and the radiation factor reduces to

$$F_3(\epsilon_e, \epsilon) = \frac{(\epsilon^2 - \epsilon_e)(3\epsilon_e - 1) + 4\epsilon_e(\epsilon_e - \epsilon)}{\epsilon^2(\epsilon_e - 1)} - \frac{\sqrt{\epsilon_e}}{2\epsilon^2}\left[2(\epsilon^2 + \epsilon_e - 2\epsilon) + (\epsilon^2 - \epsilon_e)\left(1 + \frac{1}{\epsilon_e}\right)\right] \cdot \log \frac{\sqrt{\epsilon_e} + 1}{\sqrt{\epsilon_e} - 1}. \tag{8}$$

In Fig. 3 $F_3(\epsilon_e, \epsilon)$ is plotted versus ϵ_e over a range of ϵ which extends from $\epsilon = 2\epsilon_e - 1$, for a very narrow strip (curve N), to $\epsilon = \epsilon_e$, for a very wide strip (curve W). Between these two extremes, i.e., curves N and W, F_3 is plotted again for different values of ϵ. Three important features can clearly be seen from the graph. First, in this particular case, the modified formula gives a smaller value than the original, with differences of up to 50 percent. These differences decrease slowly as ϵ_e increases. This first feature is not obvious due to the fact that E_θ vanishes in the limiting case when $\epsilon = \epsilon_e$, and hence less radiation is expected from the original formula. The second feature can be seen upon comparing the magnitude of $F_1(\epsilon_e, \epsilon)$ (the radiation factor of the open-circuit arrangement of [3]) and $F_3(\epsilon_e, \epsilon)$. It is found that the short-circuit arrangement radiates less than a quarter that of the open circuit. This fact can be utilized in the design of higher Q resonators. The third important feature that can be seen clearly in Fig. 2 is that $F_3(\epsilon_e, \epsilon)$ has a tendency to have a minimum as the strip width narrows and ϵ_e becomes less than ϵ. There is no obvious explanation for this curious behavior at this point. However, it might be that this minimum occurs because of a certain cancellation between the polarization and the strip currents. Finally, (8) possesses the asymptotic expansion

$$F_3(\epsilon_e, \epsilon) = \frac{16}{15\epsilon_e^2} + \frac{8}{3\epsilon^2} - \frac{8}{3\epsilon\epsilon_e} + \frac{48}{35\epsilon_e^3} - \frac{32}{\epsilon_e\epsilon^2} + \cdots \tag{9}$$

and, as in the previous cases, reduces to the leading term of the unmodified formula for large ϵ_e. In the limiting case of air-filled striplines, where $\epsilon = \epsilon_e \to 1$, the guidance decreases and hence spurious radiation increases. The limits of F_1 (of [3]), F_2, and F_3 (as $\epsilon = \epsilon_e \to 1$) were found to be equal to 2, 1, 2, respectively.

A similar calculation can be made for the general parallel impedance or post arrangement. Following exactly Lewin's course of calculation [1], after setting ϵ for ϵ_e in the dielectric polarization term, the modified electric field components are found to be the same as equations (6) and (7) apart from a multiplying factor R. Consequently, equations (23) and (24) of [1] still apply, using F_3 of this paper to replace F_4 of [1].

III. Conclusions

This study gives qualified approval to the continued use of Lewin's method for the calculations of microstrip radiation until a more advanced theory becomes available. Upon considering the detailed investigations of all the previous arrangements, Lewin's original results are found to be most accurate for large values of the relative effective dielectric constant ϵ_e. More accurate formulas have been presented and all found to agree with Lewin's unmodified formulas in the limiting case $\epsilon = \epsilon_e$. The short-circuit arrangement has been found to have a very valuable feature, namely, a substantially reduced amount of radiation. This feature can be utilized in the design of higher Q microstrip resonators. Finally, the general parallel post arrangement has been considered and modified formulas were derived.

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